IMPROVED 3D FEM PROGRAM FOR SIMULATING RETINAL DETACHMENT OPERATION ON AN EYEBALL

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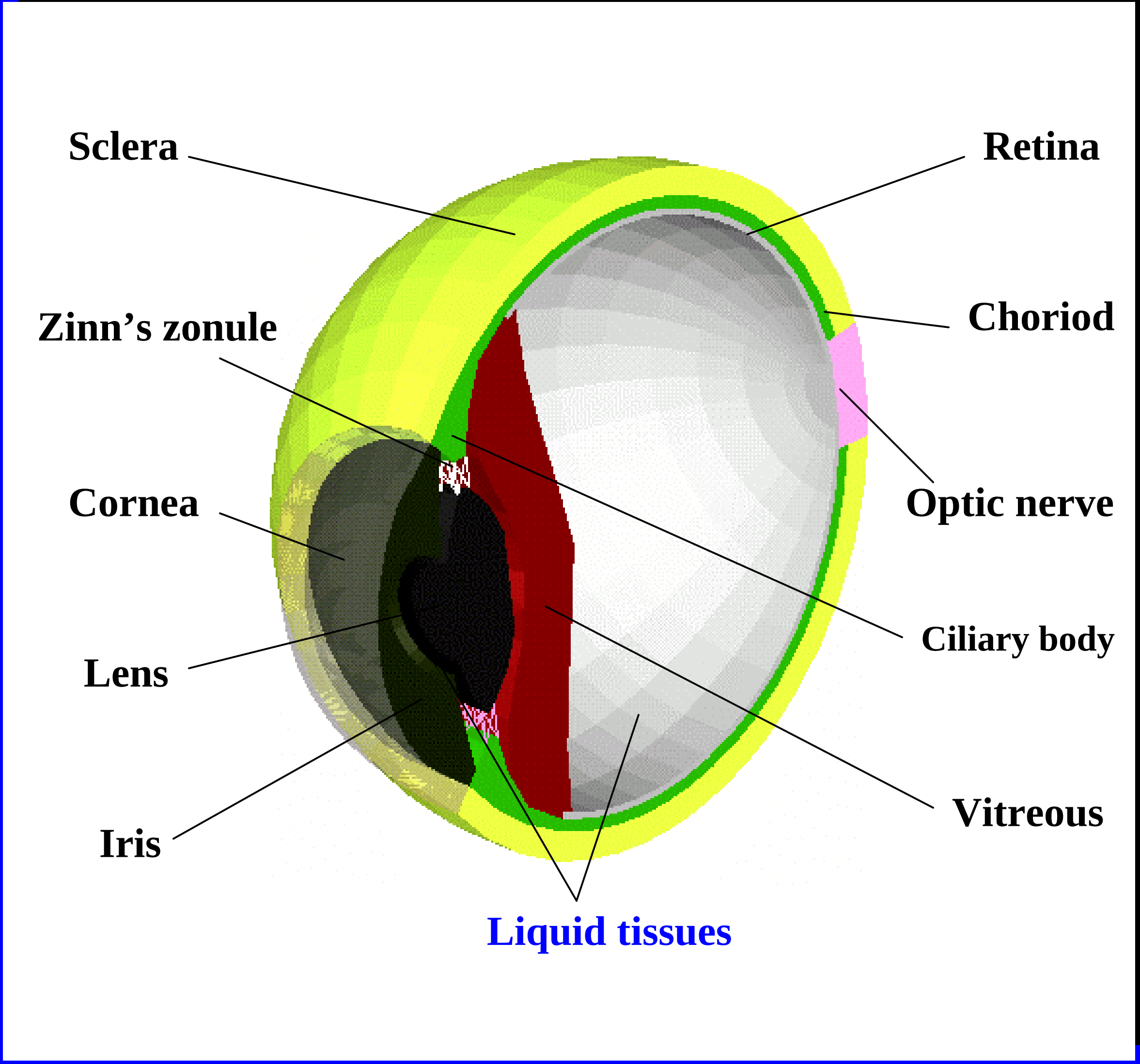
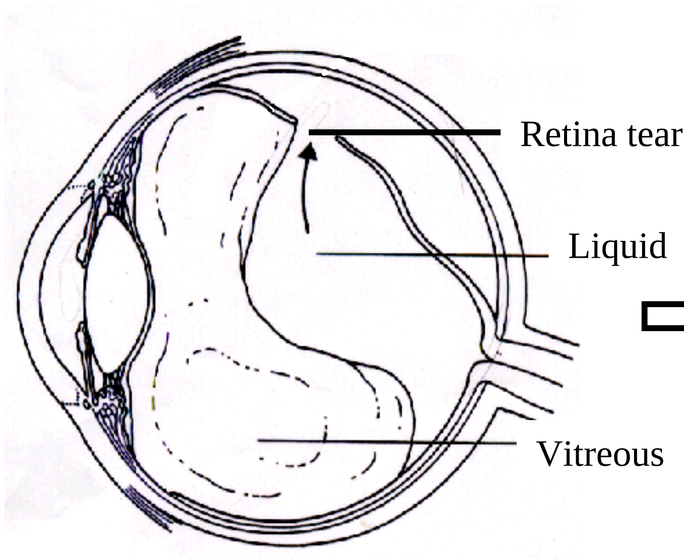
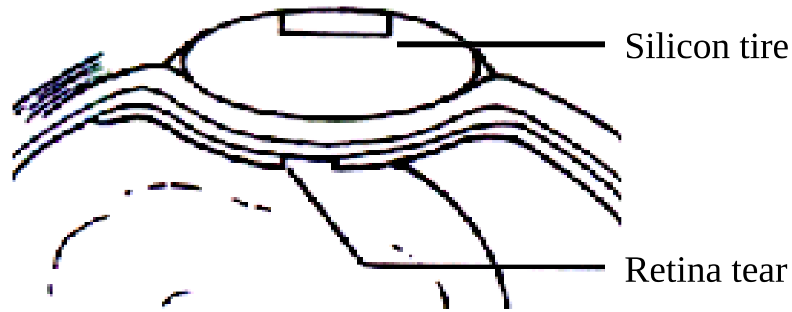
STRUCTURE OF AN EYEBALL



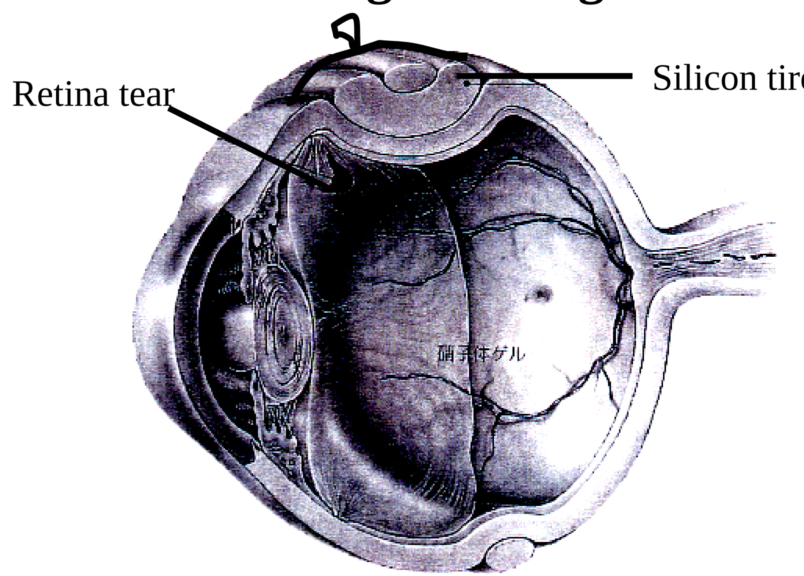
BUCKLING OPERATION





Segmental buckling

Encircling buckling



3D FEM PROGRAM FOR SIMULATION OF RETINAL DETACHMENT OPERATION ON EYEBALL

- Available element type: 3D hexahedral mixed element
  with eight displacement nodes and one pressure node.
- Available boundary conditions: nodal displacement, nodal force and pressure.
- Coupling analysis for incompressible hyperelastic solid and static liquid.

  ↑ **New algorithm**

- Available strain energy function for hyperelastic solid: $W = \sum_{r,s\geq 0} c_{rs}(I_1 - 3)^r (I_2 - 3)^s$
- Contact treatment between the deformed body and rigid body.
- **Contact treatment between the deformed bodies**
- **Suture treatment**

NUMERICAL METHOD

- New Algorithm for Solid-Liquid Coupling Analysis
- Contact Treatment Between Deformed Bodies
- Suture Treatment

New Algorithm for Solid-Liquid Coupling Analysis

Total potential energy functional for incompressible hyperelastic solid:

$$\Phi = \int_{V0} [W(\bar{I}_1, \bar{I}_2) + 2\lambda(J-1)]dV - g(u)$$

Variation of potential energy:

$$\delta\Phi = \int_{V0} [\partial W / \partial\varepsilon_{ij} + 2\lambda(\partial J / \partial\varepsilon_{ij})]\delta\varepsilon_{ij} dV + \int_{V0} 2(J-1)\delta\lambda dV - (\partial g / \partial u)\partial u = 0$$

Discretized equation for an element:

$$\begin{cases} \int_{V0} [\partial W / \partial\varepsilon_{ij} + 2\lambda(\partial\lambda / \partial\varepsilon_{ij})][(\partial\phi_N / \partial X_j)u_{Nk} + \delta_{jk}](\partial\phi_M / \partial X_i)dV = r_{Mk} \\ \int_{V0} \varphi_L 2(J-1)dV = 0 \end{cases}$$

I_1, I_2: reduced invariants of the right Cauch-Green deformation tensor

λ: Lagrange multiplier

J: determinant of the Jacobian matrix

V_0: volume of the deformed body in the reference configuration

$g(u)$: potential energy of the external force

ε_{ij}: Green-Lagrange strain

u_{Nk}: node displacement

X: initial node coordinate

ϕ, Ψ: interpolation functions of displacement and Lagrange multiplier

r_{Mk}: equivalent nodal force

New Algorithm for Solid-Liquid Coupling Analysis

Liquid pressure at current configuration:

$$P^L = P^L_{ini} + \Delta P^L = P^L_{ini} - K\frac{V^L - V^L_{ini}}{V^L_{ini}} = P^L(u^{SLI}_{11}, u^{SLI}_{12}, u^{SLI}_{13}, \bullet\bullet\bullet, u^{SLI}_{k}, u^{SLI}_{k}, u^{SLI}_{k})$$

Nodal force equivalent to liquid pressure:

$$\boldsymbol{r}^L_e = \int_{S0} \boldsymbol{\Phi}^T P^L J \boldsymbol{F}^{-T} \boldsymbol{n}_0 dS \qquad J = det|\boldsymbol{F}|$$

Discretized equation of an element for solid-liquid coupling analysis:

$$\begin{cases} \int_{V0} [\partial W/\partial \varepsilon_{ij} + 2\lambda(\partial \lambda/\partial \varepsilon_{ij})][(\partial \phi_N/\partial X_j)u_{Nk} + \delta_{jk}](\partial \phi_M/\partial X_i)dV = r^O_{Mk} + r^L_{Mk} \\ \int_{V0} \varphi_L 2(J-1)dV = 0 \end{cases}$$

$\boldsymbol{\Phi}$: 3✕12 Matrix, $\boldsymbol{F}$: deformation gradient, $\boldsymbol{n}_0$: inward normal, s_0 : element area,
P^L: liquid pressure, P_{ini}: initial liquid pressure, V_{ini}: initial liquid volume.

New Algorithm for Solid-Liquid Coupling Analysis

Previous algorithm

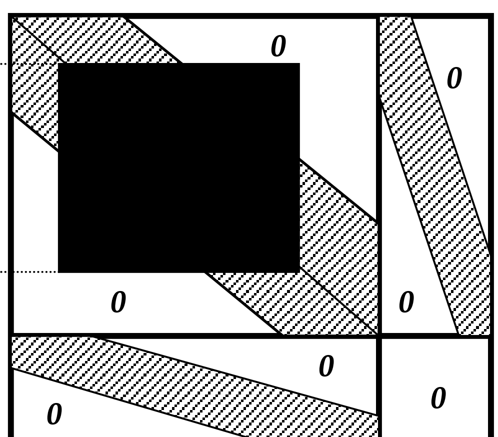
$$
\begin{cases}
f_{ij}(u_{11},u_{12},u_{13},\cdots,u_{n1},u_{n2},u_{n3},\lambda_1,\cdots,\lambda_m)=r_{ij}^{O} \\
f_{ij}(u_{11},u_{12},u_{13},\cdots,u_{n1},u_{n2},u_{n3},\lambda_1,\cdots,\lambda_m)=r_{ij}^{O}+ \\
\qquad \underline{r_{ij}^{L}(P^{L}(u_{11}^{SLI},u_{12}^{SLI},u_{13}^{SLI},\cdots,u_{k1}^{SLI},u_{k2}^{SLI},u_{k3}^{SLI}))} \\
f_{3n+l}(u_{11},u_{12},u_{13},\cdots,u_{n1},u_{n2},u_{n3})=0
\end{cases}
$$

(i=1,n j=1,3 l=1,m)

$$
\begin{bmatrix} K_u^{i-1}+K_{1\ L}^{i-1} & K_\lambda^{i-1} \\ K_\lambda^{i-1\,T} & 0 \end{bmatrix}
\begin{bmatrix} \Delta U^i \\ \Delta\lambda^i \end{bmatrix}
=
\begin{bmatrix} FU^{i-1} \\ F\lambda^{i-1} \end{bmatrix}
+
\begin{bmatrix} R+R_L^{i-1} \\ 0 \end{bmatrix}
$$

3(I-1)+1

3(I+k-1)+3

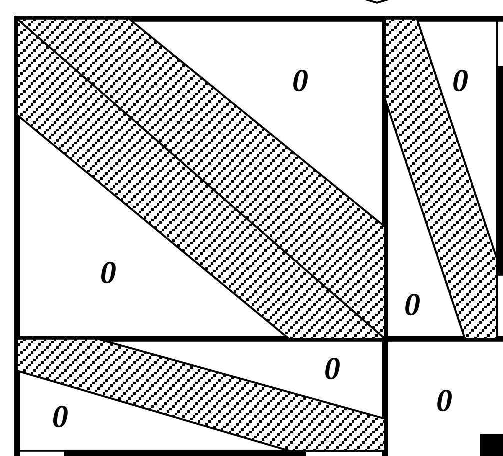


Nonzero term increased by introducing liquid ■

New algorithm

$$
\begin{cases}
f_{ij}(u_{11},u_{12},u_{13},\cdots,u_{n1},u_{n2},u_{n3},\lambda_1,\cdots,\lambda_m)=r_{ij}^{O} \\
f_{ij}(u_{11},u_{12},u_{13},\cdots,u_{n1},u_{n2},u_{n3},\lambda_1,\cdots,\lambda_m)=r_{ij}^{O}+\underline{r_{ij}^{L}(P^{L})} \\
f_{3n+l}(u_{11},u_{12},u_{13},\cdots,u_{n1},u_{n2},u_{n3})=0 \\
p^{L}-(p_{ini}^{L}+\Delta p^{L}(u_{11}^{SLI},u_{12}^{SLI},u_{13}^{SLI},\cdots,u_{k1}^{SLI},u_{k2}^{SLI},u_{k3}^{SLI}))=0
\end{cases}
$$

$$
\begin{bmatrix} K_u^{i-1} & K_\lambda^{i-1} & K_p^{i-1} \\ K_\lambda^{i-1\,T} & & \\ K_p^{i-1\,T} & 0 & \end{bmatrix}
\begin{bmatrix} \Delta U^i \\ \Delta\lambda^i \\ \Delta P^i \end{bmatrix}
=
\begin{bmatrix} FU^{i-1} \\ F\lambda^{i-1} \\ FP^{i-1} \end{bmatrix}
+
\begin{bmatrix} R \\ 0 \\ 0 \end{bmatrix}
$$



3(I-1)+1

3(I+k-1)+3

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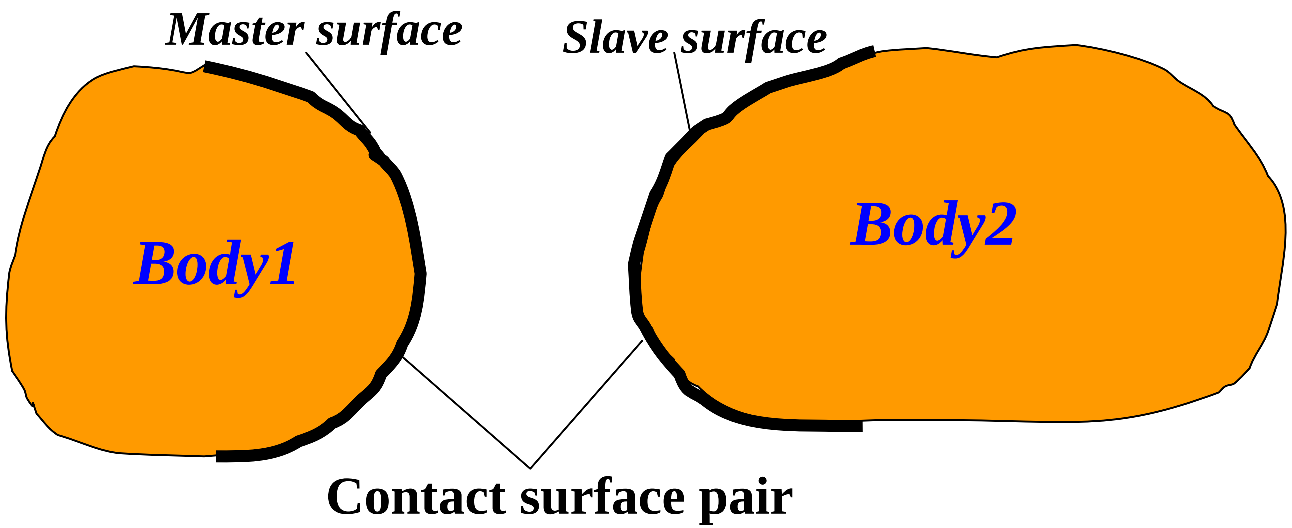
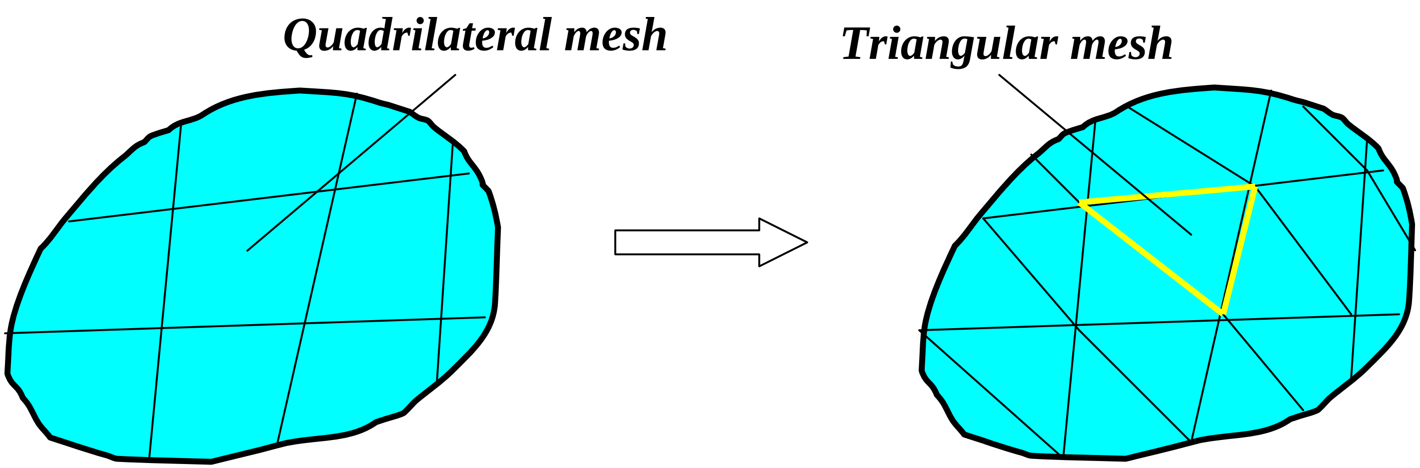
NUMERICAL METHOD

- New Algorithm for Solid-Liquid Coupling Analysis
- Contact Treatment Between Deformed Bodies
- Suture Treatment

Contact Treatment Between Deformed Bodies

Specification and description of contact surface





Description of the master surface

Contact Treatment Between Deformed Bodies

Contact treatment formulation

Total potential energy function
for contact treatment(penalty method):

$$\Phi_P = \Phi + \pi_P$$

$$\pi_P = \frac{1}{2}\alpha G^2$$

Penetration of hitting node into target segment:

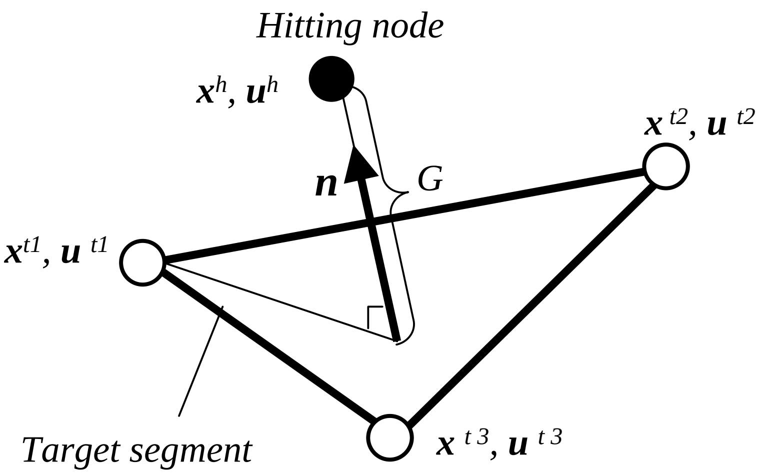
$$G = (\boldsymbol{x}^h - \boldsymbol{x}^{t1}) \cdot \boldsymbol{n}$$

Variation of total potential energy:

$$\delta\Phi_P = \delta\Phi + \delta\pi_p = \delta\Phi + \delta\boldsymbol{u}_c^T \cdot (\alpha G\boldsymbol{D}) = \delta\Phi + \delta\boldsymbol{u}_c^T \cdot \boldsymbol{f}_c$$

$\boldsymbol{\alpha}$: *penalty constant*
$\boldsymbol{f}c$: contacting force vector
$\boldsymbol{D}$: 12×1matrix
$\boldsymbol{u}$: virtual displacement vector:

$$\delta\boldsymbol{u}_c^{\ T} = (\delta u_x^h, \delta u_y^h, \delta u_z^h, \delta u_x^{t1}, \delta u_y^{t1}, \delta u_z^{t1}, \delta u_x^{t2}, \delta u_y^{t2}, \delta u_z^{t2}, \delta u_x^{t3}, \delta u_y^{t3}, \delta u_z^{t3})^T$$



Hitting node and target segment
in contact state

NUMERICAL METHOD

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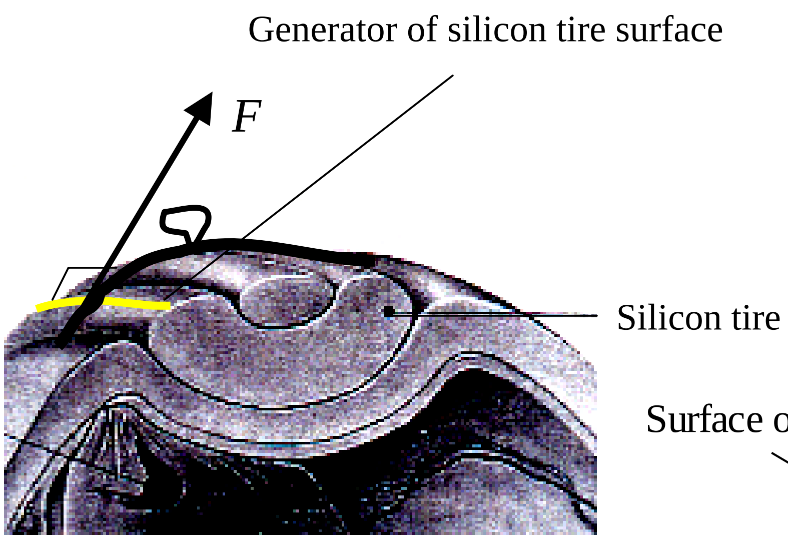
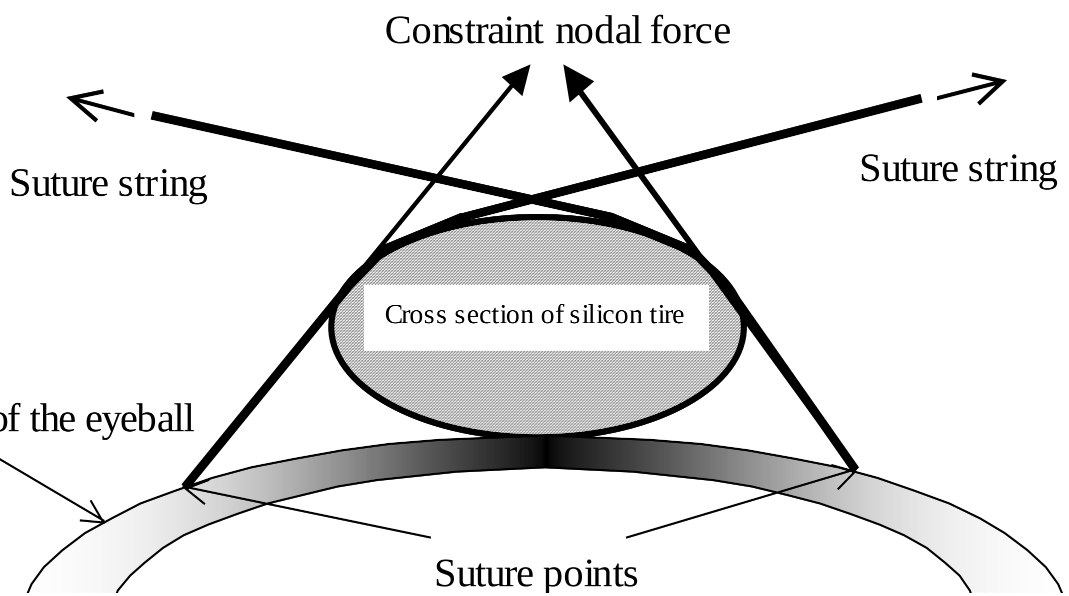
Suture Treatment



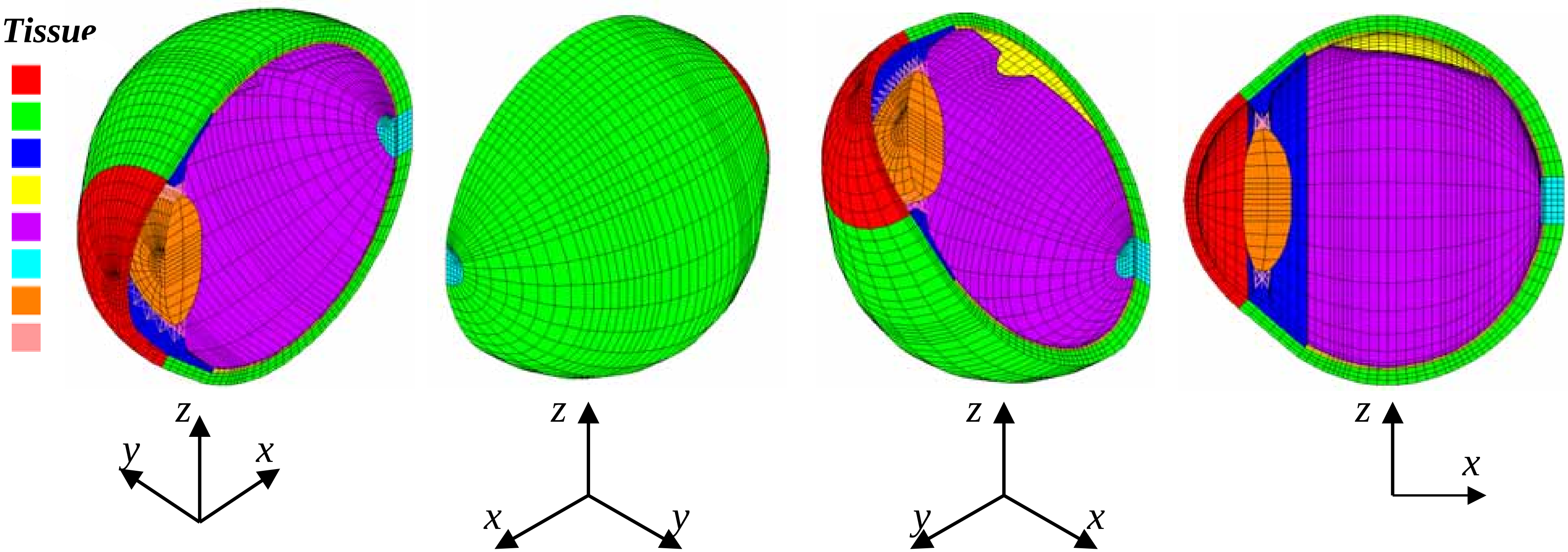


Enforcing nodal force boundary conditions to the suture points

SIMULATION
OF ENCIRCLING BUCKLING OPERATION

- Finite Element Model for Analysis
- Analysis Conditions
- Analysis Results

Finite Element Model for Analysis



9862 elements

Analysis Conditions

Material model

Soft tissues except zinn's zonule: neo-Hooke hyperelastic material model $W = c(I_1 - 3)$

Zinn's zonule: linear elastic material model

Liquid: static liquid

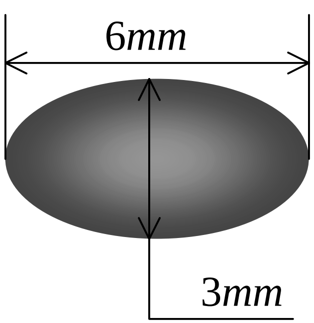
Material constants for soft tissues except zinn's zonule

	Cornea	***Sclera***	***Ciliary body***	***Choriod***	***Retina***	***Optic nerve***	***lens***
c	*0.0333*	*0.0833*	*0.01*	*0.0083*	*0.0008*	*0.05*	*10.0*

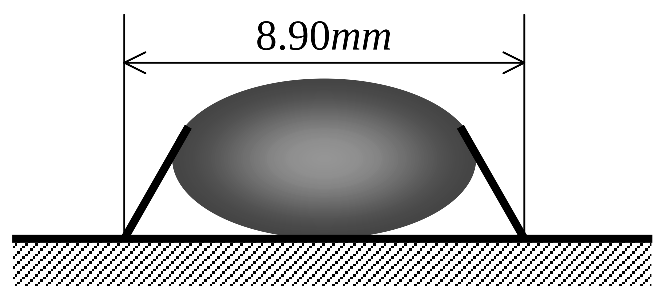
Zinn's zonule Young's modulus: 100*MPa*

Liquid buck modulus: 2083.3*MPa*

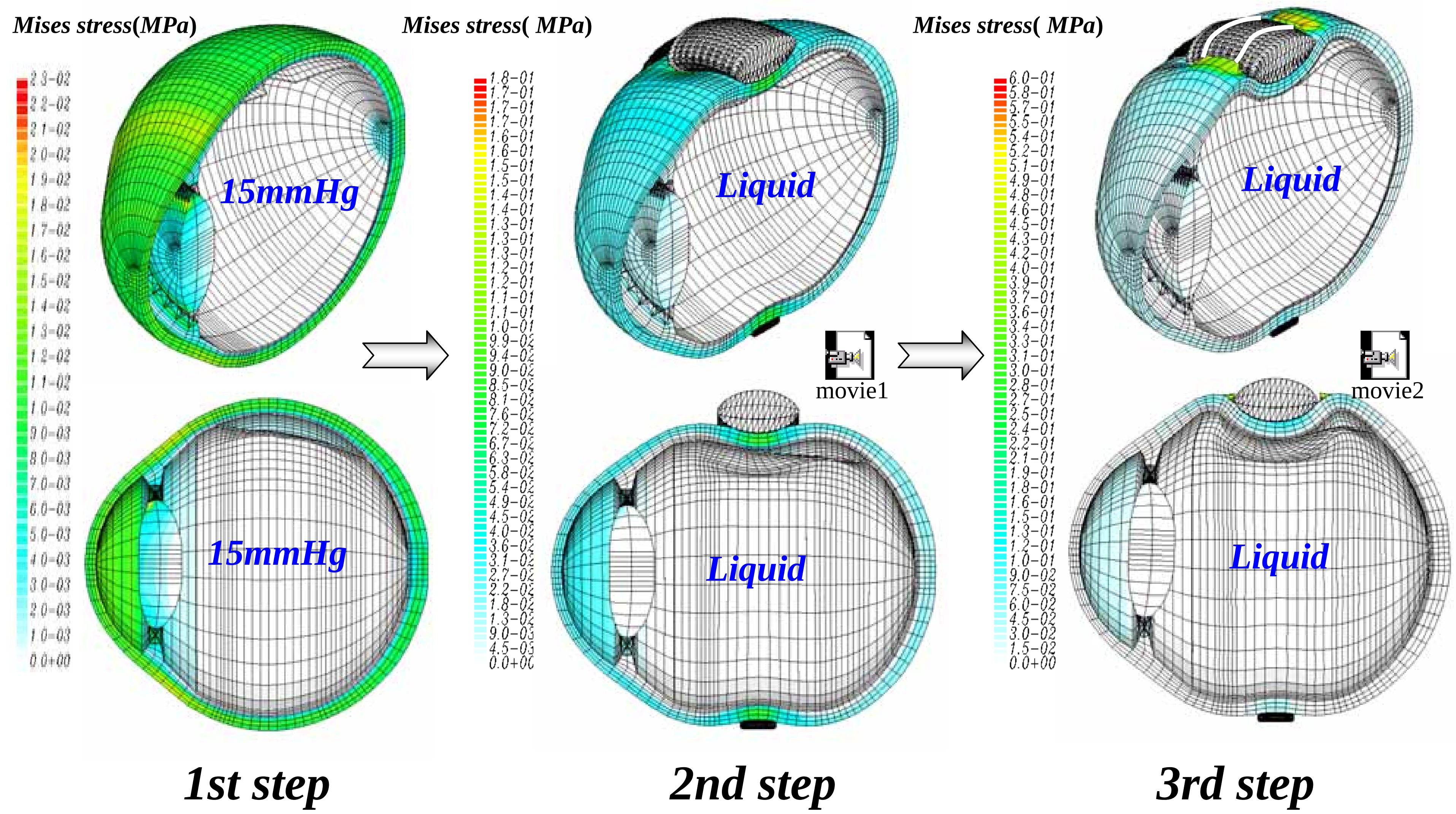
Buckle shape



Suture width



Analysis Results



CONCLUSION

- The new algorithm for the solid-liquid coupling analysis and the new functions of the contact treatment between deformed bodies and suture treatment are effective. The improved program enables practical scleral buckling operation.

- The simulation of encircling buckling operation performed using this program was successful. The results of the stress and deformation, including the change in optic axis length, the indenting effect, and the reattachment state of the retina to the choriod was reasonable. This program has the ability of predicting suitable factors for a satisfactory buckling operation.

END