1-D numerical analysis of blood flow in multi-branched arteries

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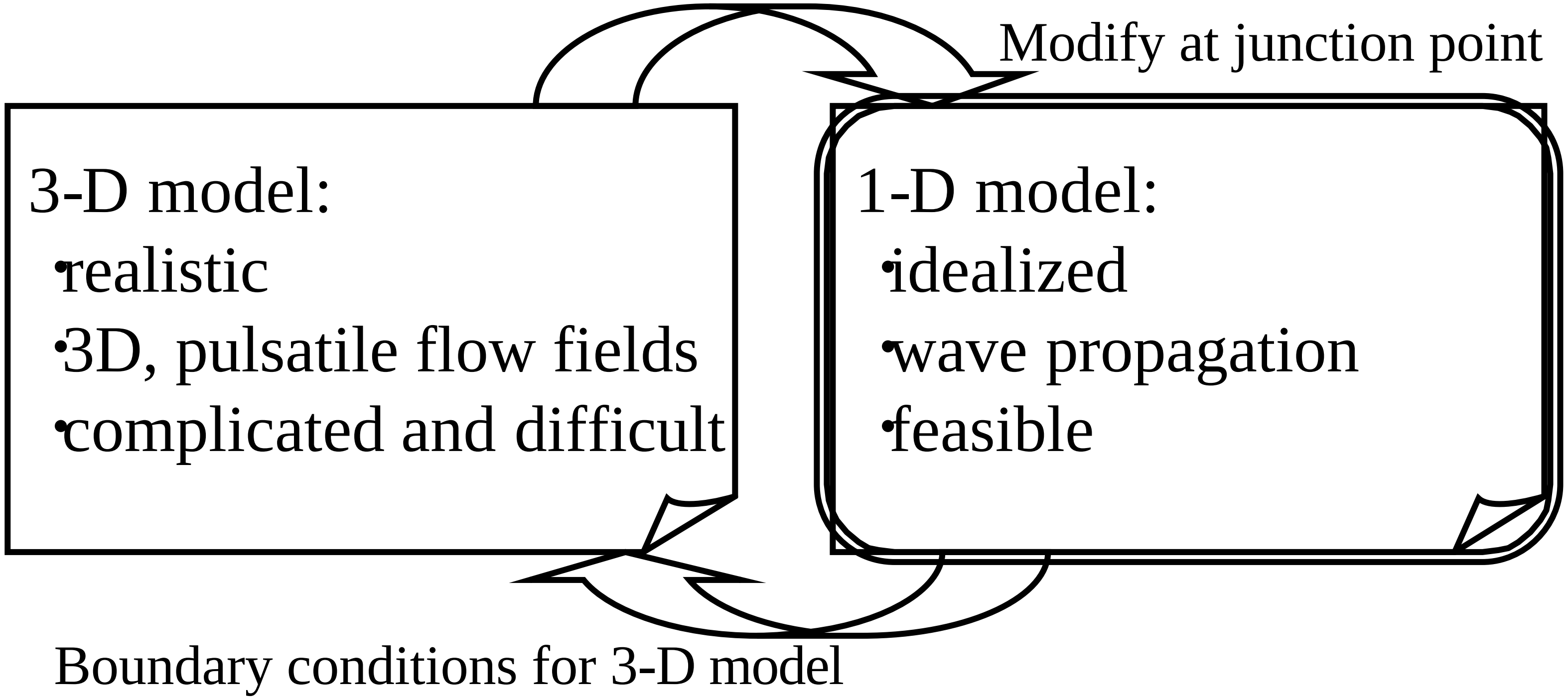
Objective

To make a computational model of whole body circulatory system



Research plan

- **Goal:**

  To make a 1-D computational model for whole body circulatory system

  – methods : analysis of the pulse wave propagation

- **Strategy**

  – establishment : 1-D model ⇐ many issues to be considered

  – verification and validation :

    - comparison with 3-D model simulation
    - comparison with experimental results
    - comparison with in vivo data

  – combination : 1-D model and 3-D model

Influence of some issues

- **Vessel structure**
  - curvature, taper, branch angle, outflow

current task

- **Unsteadiness of blood flow**
- **Behavior of the vessel wall**
  - visco-elasticity of the wall
  - effect of longitudinal tethering
- **Non-newtonian characteristics of the blood**
- **Boundary conditions**
  - inlet flow, peripheral conditions

1-D computational model

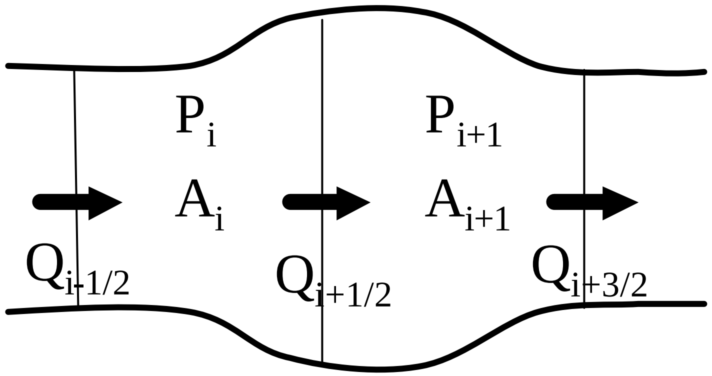
■ Modeling

– variables

P : Pressure

A : sectional area

Q : Flow Volume



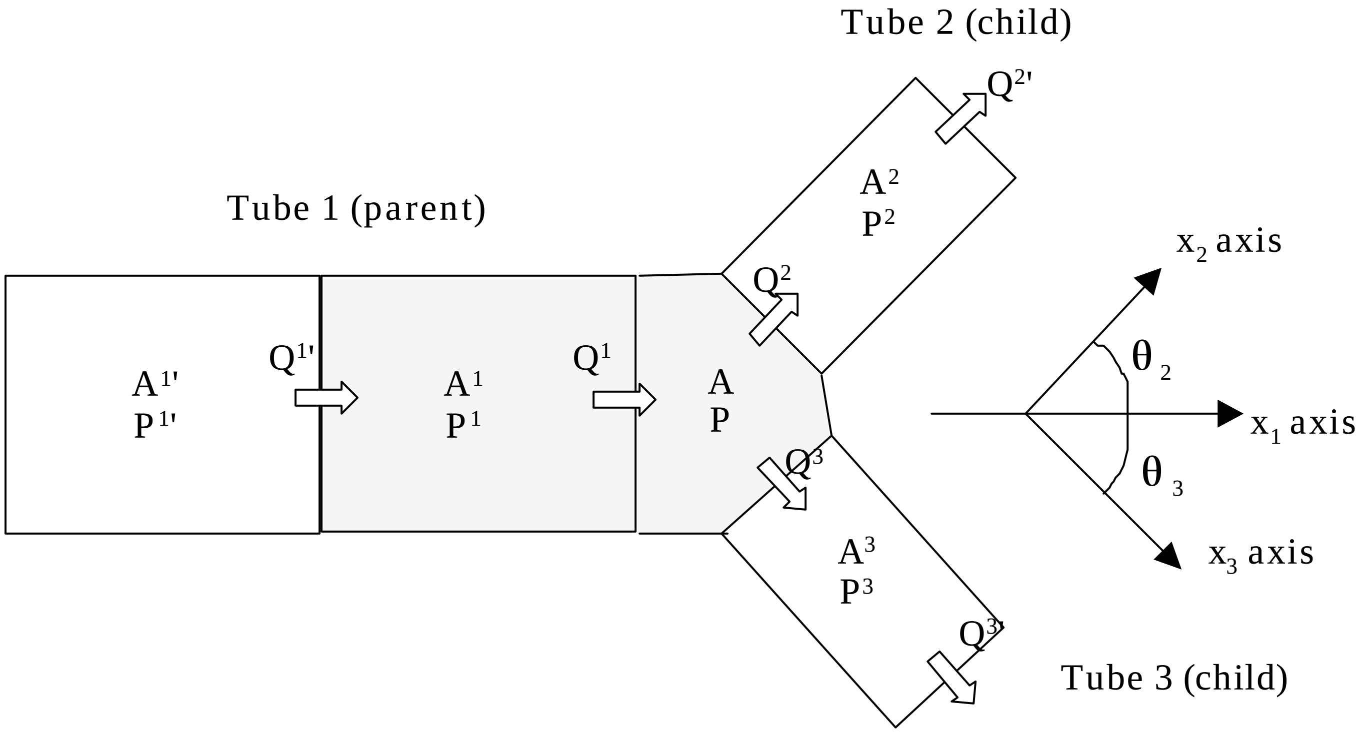
■ Governing equations

– continuity : $\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$

– momentum conservation : $\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + \frac{A}{r}\frac{\partial p}{\partial x} + \frac{8pm}{r}\frac{Q}{A} = 0$

– deformation of the tube : $p = p_0 \exp\left(\frac{1}{K}\left(\frac{A}{A_0} - 1\right)\right)$

Treatment at branching points



$$\frac{dQ^{1}}{dt} = \{\frac{AA^{1}}{A^{1}+A}(\frac{(Q^{1'})^{2}}{(A^{1'}+A^{1})/2} - \frac{(Q^{2})^{2}}{AA^{2}}\cos q_{2} - \frac{(Q^{3})^{2}}{AA^{3}}\cos q_{3}) + (P^{1}-P)AA^{1}/r\}/\Delta x$$

$$AA^{1} = (\frac{2A^{1}k^{1}}{A^{1}k^{1}+A^{2}k^{2}+A^{3}k^{3}}A + A^{1})/2$$

cross-sectional area ratio of the tubes : $\frac{\sum A_{output}}{A_{input}}$

Models

1:The diameter of the tube

- **large tube : aorta**
- **medium tube : middle artery**
- **small tube : arteriole**

2:The bifurcation angle

0°～120° (with an interval of 30°)

3:The cross-sectional area ratio of the tubes

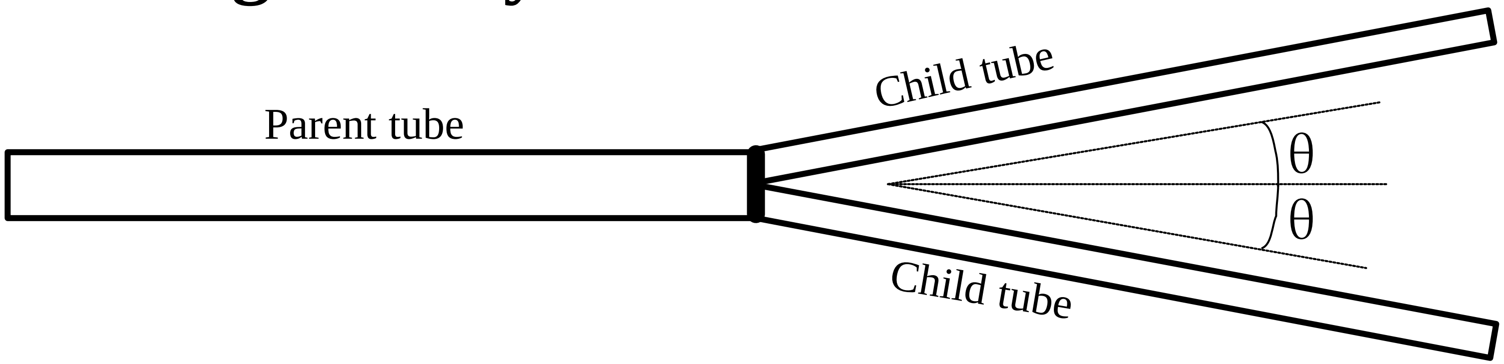
0.8～1.2 (with an interval of 0.1)

Computational parameters

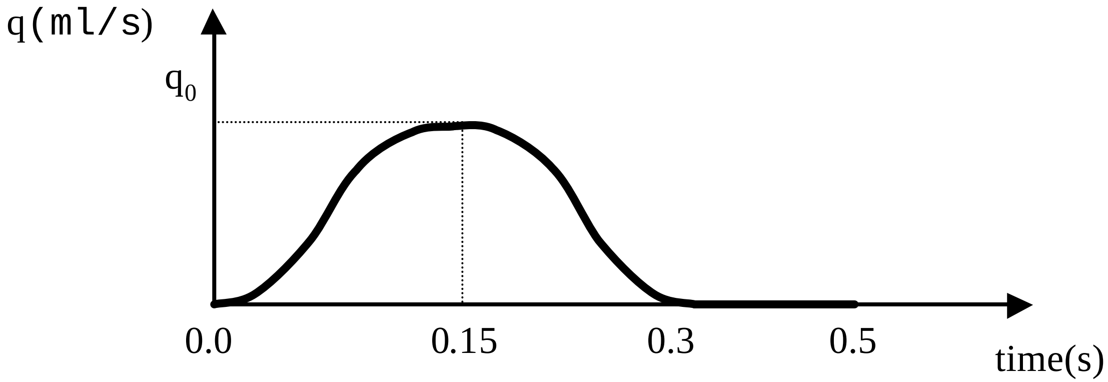
diameter	*Large*	*Medium*	*Small*
Cross-sectional area **(tube diameter)**	**5.0×10^{-4} m** **(25mm)**	**7.0×10^{-6} m** **(3mm)**	**2.0×10^{-7} m** **(0.5mm)**
Maximum flow volume (q_0) **(peak flow velocity)**	**5.0×10^{-4} m^3/s** **(100cm/s)**	**5.0×10^{-6} m^3/s** **(70cm/s)**	**3.0×10^{-8} m^3/s** **(15cm/s)**
Reynolds number (Re)	**8300**	**700**	**25**
The relation coefficient (K) **(wave propagation velocity)**	**4.0** **(5.0 m/s)**	**10.0** **(7.9 m/s)**	**25.0** **(12.5 m/s)**
Length of the tube **(△x)**	**1.0m** **(1.0mm)**	**1.5m** **(2.0mm)**	**2.0m** **(5.0mm)**
Total elapsed time **(△t)**	**0.5s** **(0.1ms)**	**0.5s** **(0.1ms)**	**0.5s** **(0.2ms)**
Courant Number (=c△t/△x)	**0.5**	**0.395**	**0.5**

A computational model

- **Model geometry**



- **Boundary conditions**



Reflected wave at the branching point

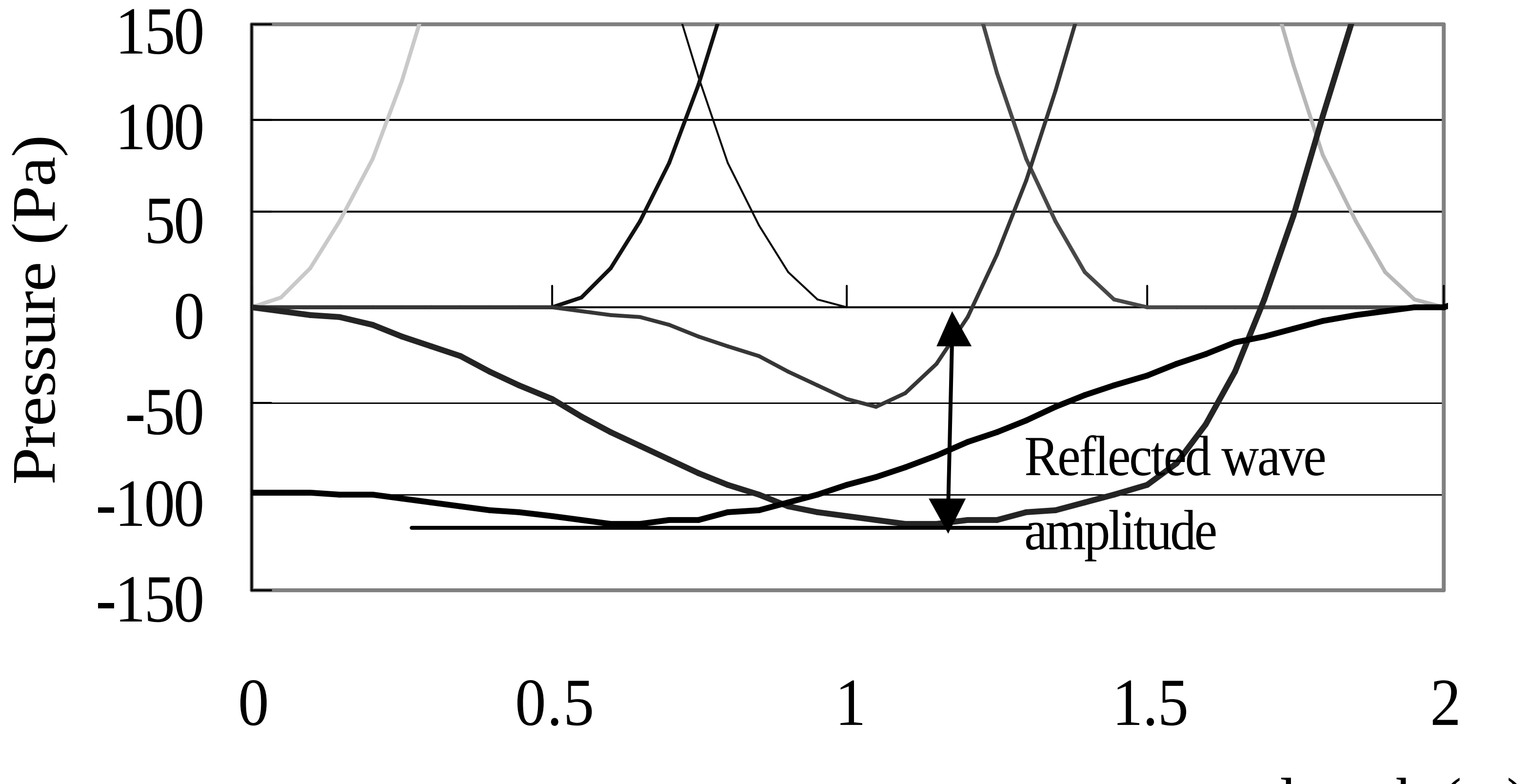


Fig. 4

Relationship between the reflected wave and the tube diameter

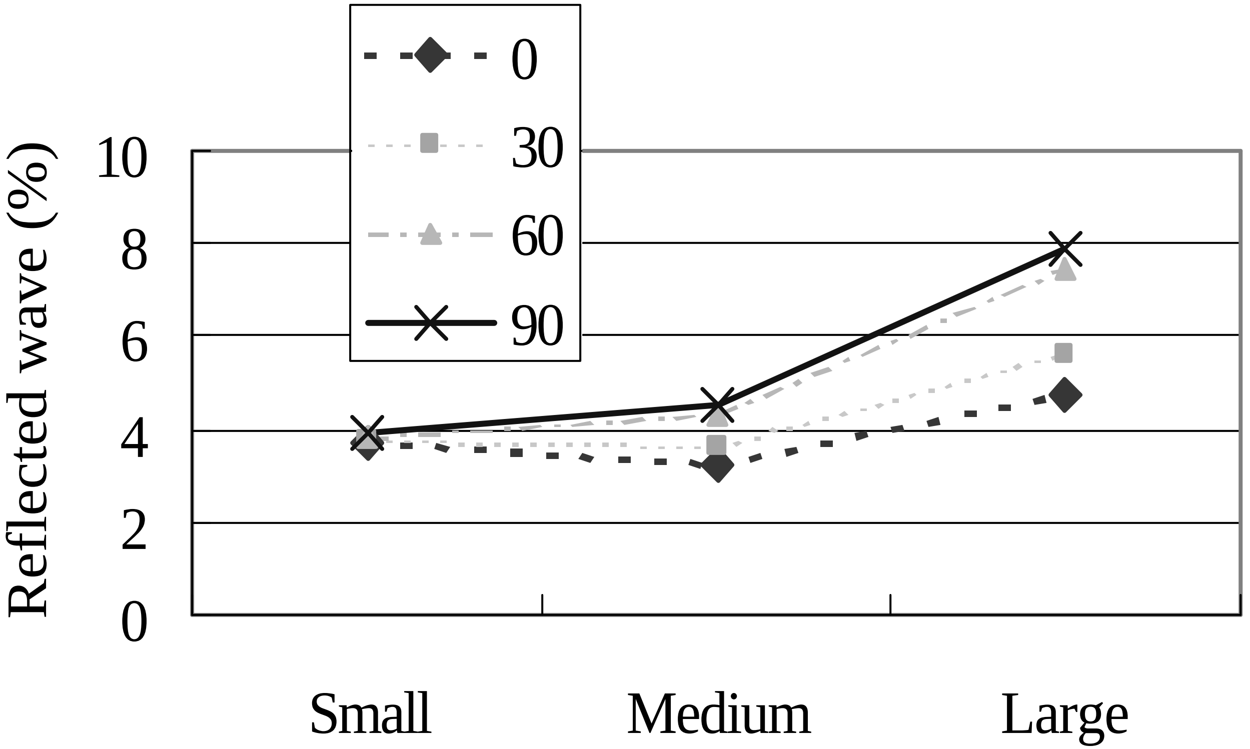


Fig. 7 : cross-sectional ratio 1.1

Dependence on bifurcation angle of the reflected wave

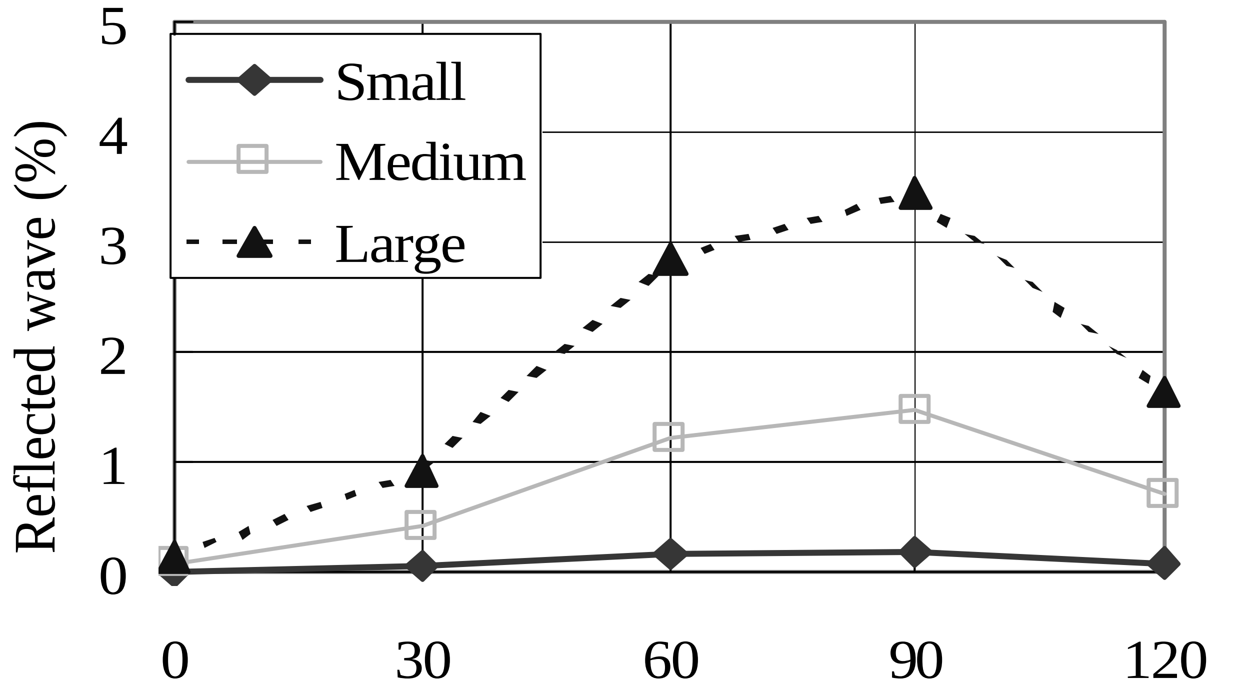


Fig. 8

Relationship between the reflected wave and the tube cross-sectional ratio

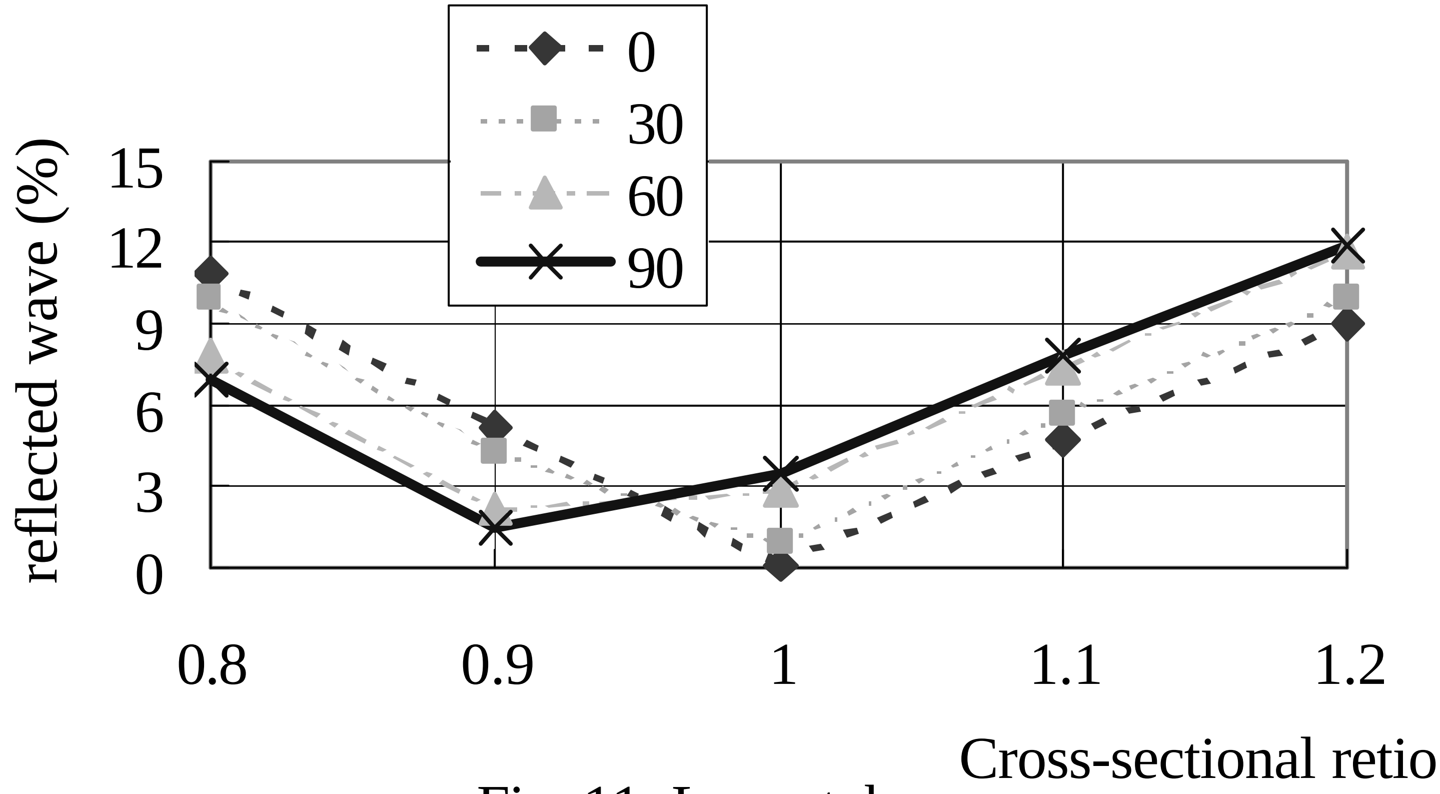


Fig. 11: Large tube

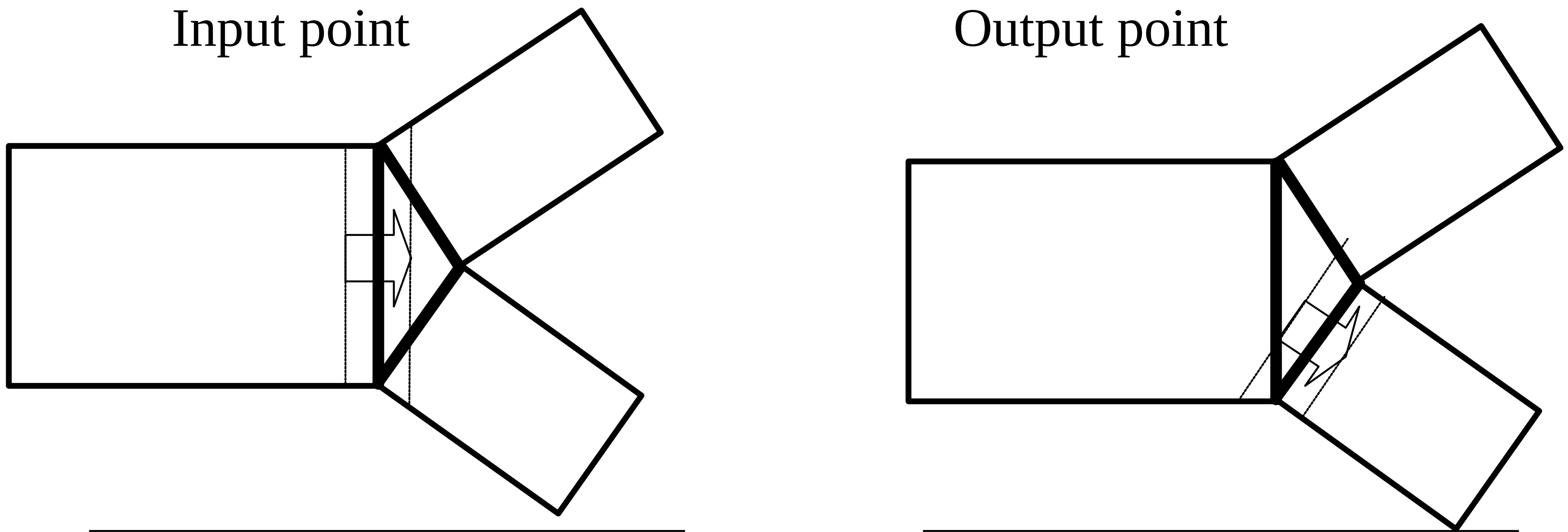
Discussion and Conclusion

- **1-D computational model of the artery systems**
  - investigation the bifurcation angle dependence
  - a quantitative analysis of the reflected wave
- **The angle effect**
  - the reflected wave at bifurcation point was observed
  - the angle dependence was recognized in large and medium arteries
- **Combination of angle and cross-sectional ratio**
  - peculiar feature of reflected wave

Future works

- **Establishment of the 1-D model**
  - **vessel structure**: curvature, taper, branch angle, outflow
  - **unsteadiness of blood flow**
  - **behavior of the vessel wall**
  - non-newtonian characteristics of the blood
  - boundary conditions
- **Verification and validation**
  - comparison with 3-D model
  - comparison with experimental results
  - comparison with in vivo data
- **Model combination : 1-D model and 3-D model**

Reflected wave at branch point



increase the cross section → negative reflected wave

decrease the cross section → positive reflected wave

→ phase difference

Total reflected wave at branch point