Numerical Study on the Relationship Between the Flow Rate and Temperature in Peripheral Arteries Simulated by A One-dimensional Model of An Elastic Tube

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- Introduction
- The one-dimensional nonlinear fluid model
- Energy equation in an elastic tube
- Computed results:

a. in a single vessel

b. in vessels with a bifurcation

- Concluding Remarks

Research Background

- Blood Circulation significantly influences body temperature. The factors to affect the blood circulation will cause the variation of body temperature (especially in the peripheral part).

→ Aging , Exercising, mental stress, Smoking etc…

- Measuring skin temperature is an important method to diagnose blood circulation illness (such as by thermography)

- What is the relationship between blood pressure, flow rate and temperature?

The previous studies

Hemodynamic models:

- Blood flow in arteries with structured –tree model (Olufsen, et al 2000)
- Blood flow in the cerebral circulation of man (Zagzoule, et al, 1986)
- Blood flow in the whole human circulation (Sheng, et al, 1995)
- The analysis of blood flow in branched arteries (Kitawaki and Himeno, 2000)
- Multi-scale model of blood flow (Liu, 2002)

Thermal models:

- Keller and Seiler's model (1971)
- Human thermal model (Takemori, et al, 1995)

One-dimensional thermo-fluid model in an elastic blood vessel

Continuity equation

$$\frac{\partial A}{\partial t}+\frac{\partial q}{\partial x}=0$$

momentum equation

$$\frac{\partial q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{q^2}{A}\right)+\frac{A}{\rho}\frac{\partial P}{\partial x}=-\frac{2\pi \nu R}{\delta}\frac{q}{A}$$

state equation

$$P(x,t)-P_0=\frac{4}{3}\frac{Eh}{r_0}\left(1-\sqrt{\frac{A_0}{A}}\right)$$

The equation by substituting state equation into momentum equation

$$\frac{\partial q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{q^2}{A}+B\right)=-\frac{2\pi\nu R}{\delta}\frac{q}{A}+C$$

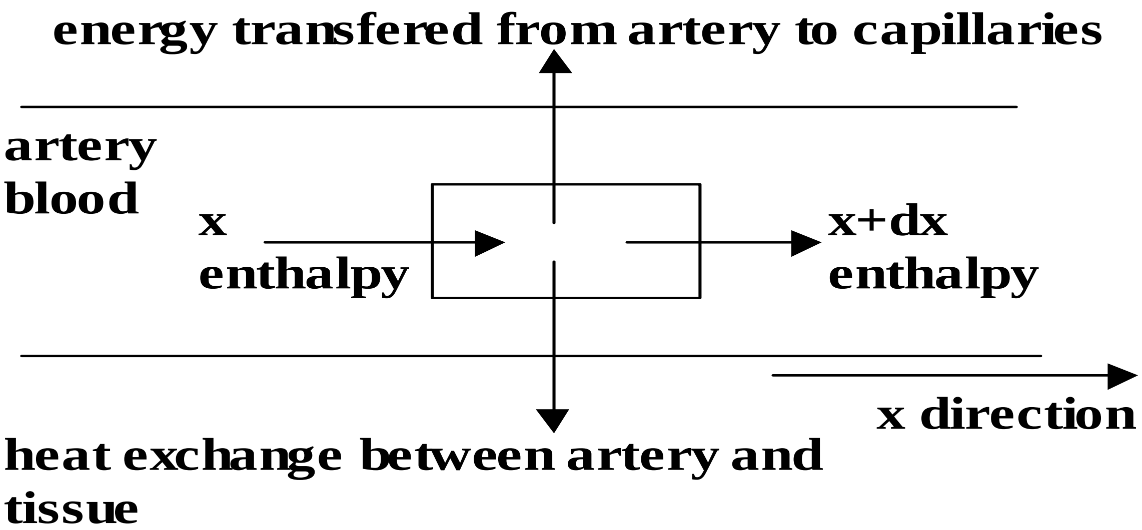
$$B=\sqrt{\pi A}\frac{1}{\rho}\frac{4}{3}Eh$$

$$C=\sqrt{\pi A}\frac{8}{3}\frac{1}{\rho}\frac{\partial}{\partial x}(Eh)-\frac{4}{3}\frac{A}{\rho}\frac{\partial}{\partial x}\left(\frac{Eh}{r_0}\right)$$

Energy equation

$$\frac{\partial(AT_a)}{\partial t}+\frac{\partial(qT_a)}{\partial x}=-\omega AT_a-\frac{(hA_s)}{\rho_b c_b}(T_a-T_t)$$

The Derivation of the Energy Equation



Energy Balance equation in Arteries

$$\frac{\partial(\rho_b c_b A T_a)}{\partial t} = -\frac{\partial(\rho_b c_b u A T_a)}{\partial x} - \omega \rho_b c_b A T_a - h A_s (T_a - T_t)$$

Initial and Boundary Conditions

Initial condition

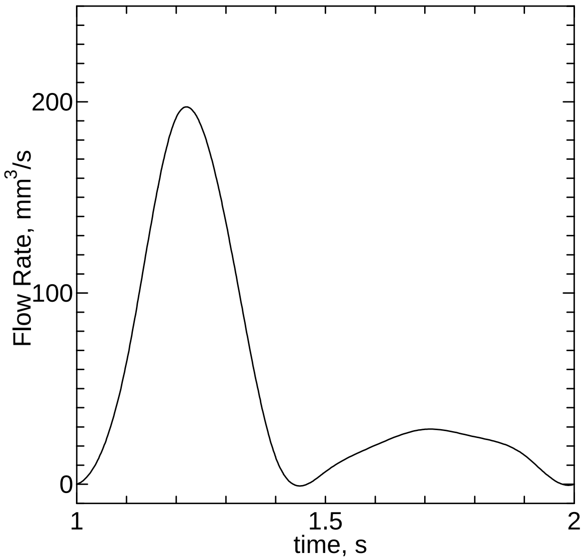
$$P = P_0 \qquad q = 0 \qquad A = A_0 \qquad T = T_0$$

Inflow condition

- flow rate in physiological form (Mcdonald,1974)

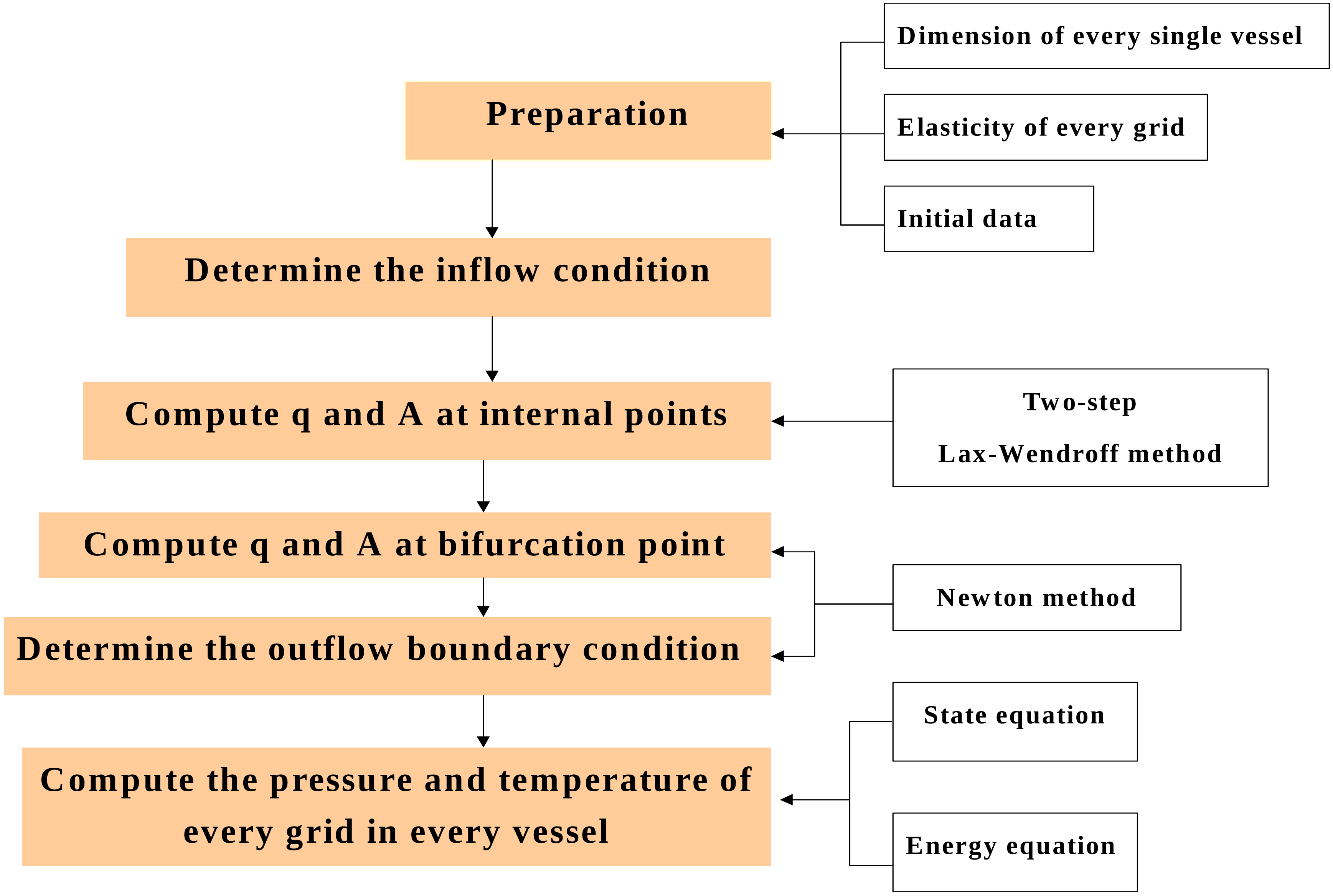
$$q_{in} = q_{\max}(0.251 + 0.290(\cos\Phi + 0.97\cos 2\Phi + 0.47\cos 3\Phi + 0.14\cos 4\Phi)$$

$$\Phi = 2\pi t - 1.4142$$



Outflow condition

$$p(x,t) - p_0 = R_T q(x,t)$$


Dimension of every single vessel
Preparation
Elasticity of every grid
Initial data
Determine the inflow condition
Compute q and A at internal points
Two-step
Lax-Wendroff method
Compute q and A at bifurcation point
Newton method
Determine the outflow boundary condition
State equation
Compute the pressure and temperature of every grid in every vessel
Energy equation

Internal boundary conditions at bifurcation points

$q_p^{m+1} \quad q_{d1}^{m+1} \quad q_{d2}^{m+1}$

(1)

Continuity equation

(2)

$q_p^{m+1} = q_{d1}^{m+1} + q_{d2}^{m+1}$

(3)

The function of A_n^{m+1}

(5)

(2)

(4) Newton method

$p_p^{m+1} = p_{d1}^{m+1} = p_{d2}^{m+1}$

(6)

A_n^{m+1}

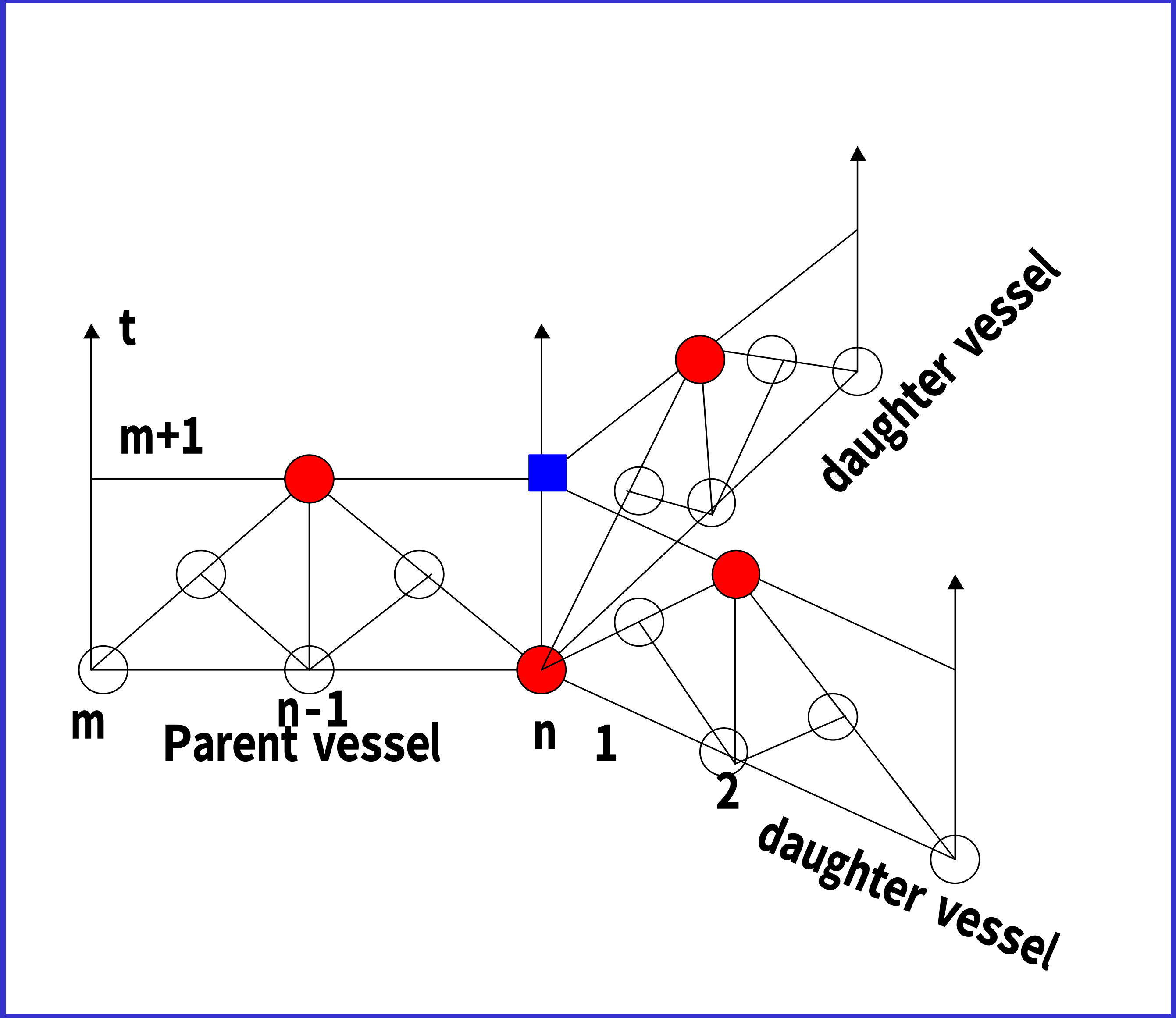
$A_{1d1}^{m+1} \quad A_{1d2}^{m+1}$

Energy equation

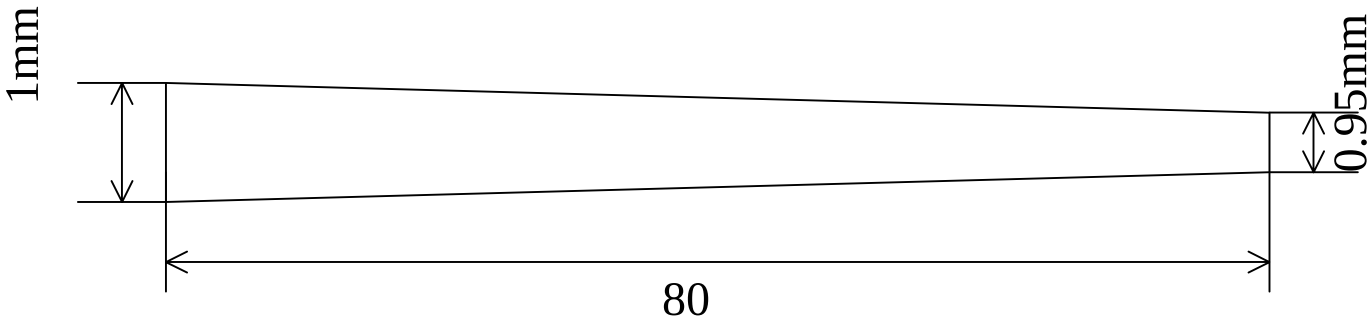
$T_p^{m+1} = T_{d1}^{m+1} = T_{d2}^{m+1}$

$T_{1d1}^{m+1} \quad T_{1d2}^{m+1}$

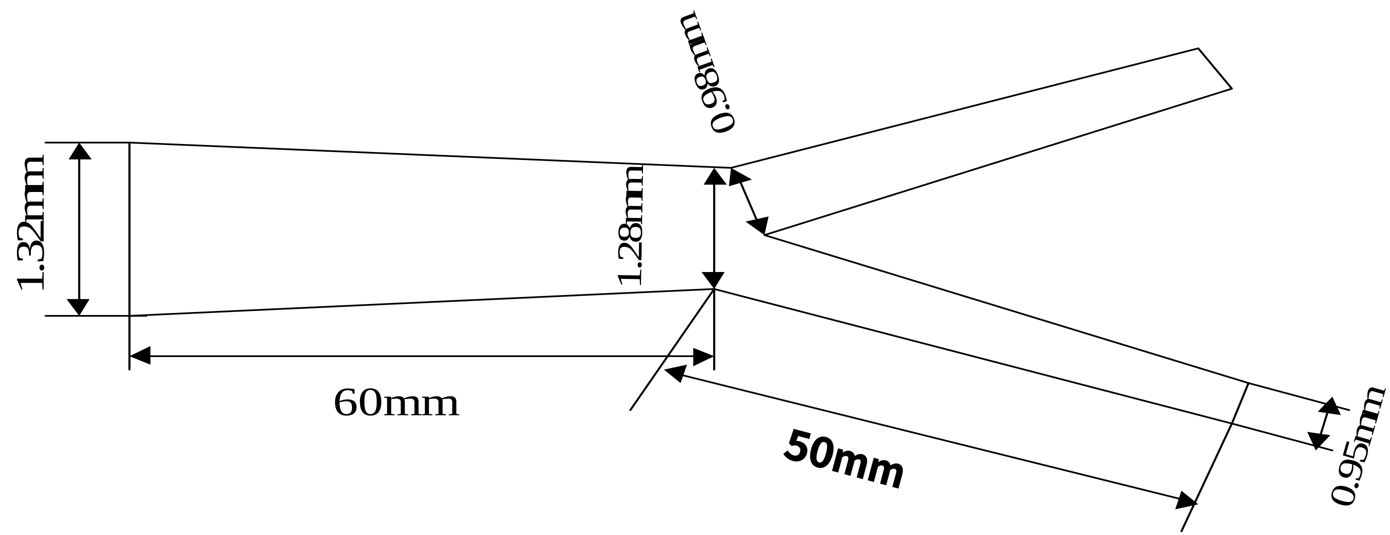
Method to determine the parameters at the bifurcation point

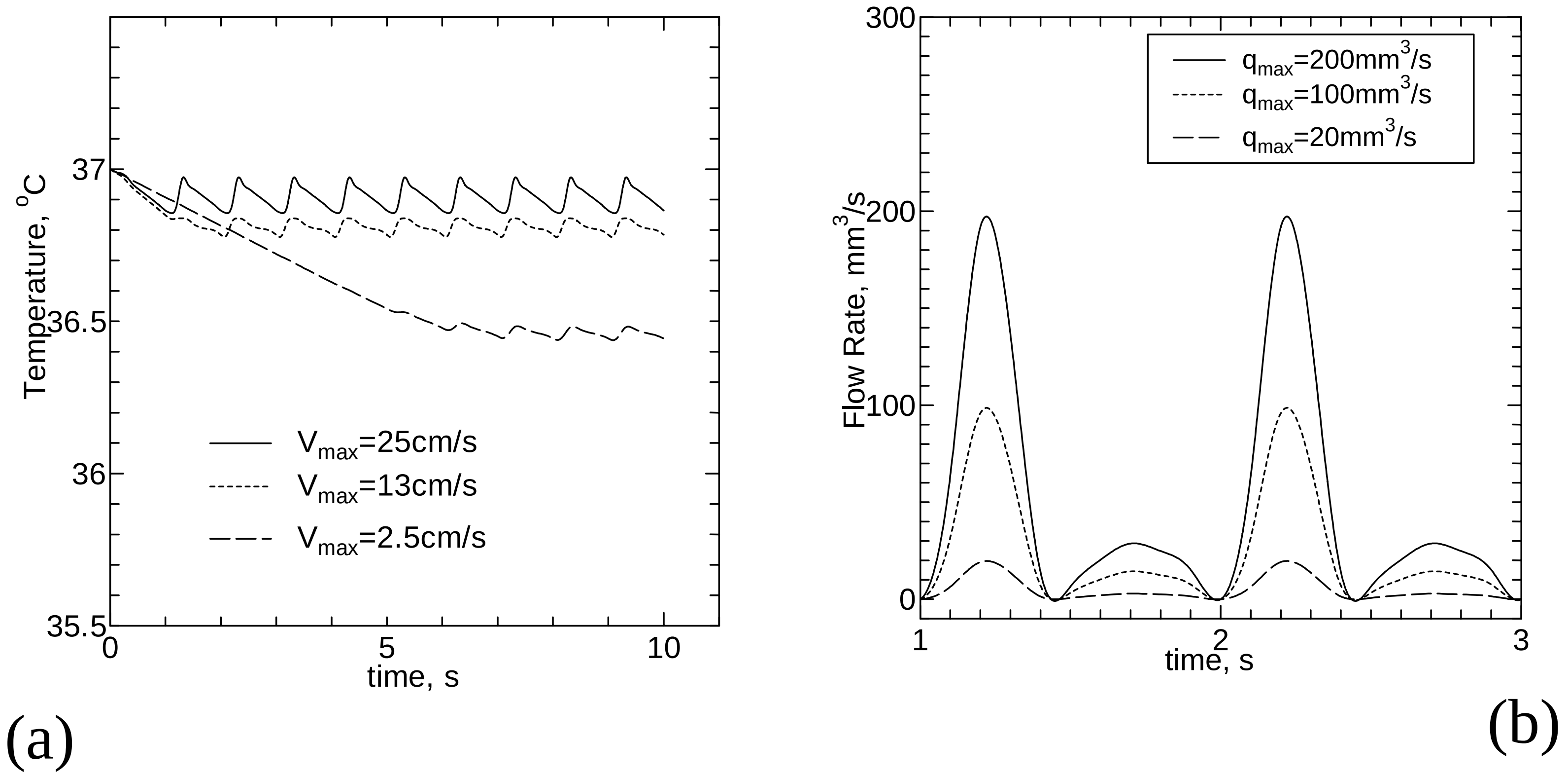


A simulated single arterial vessel



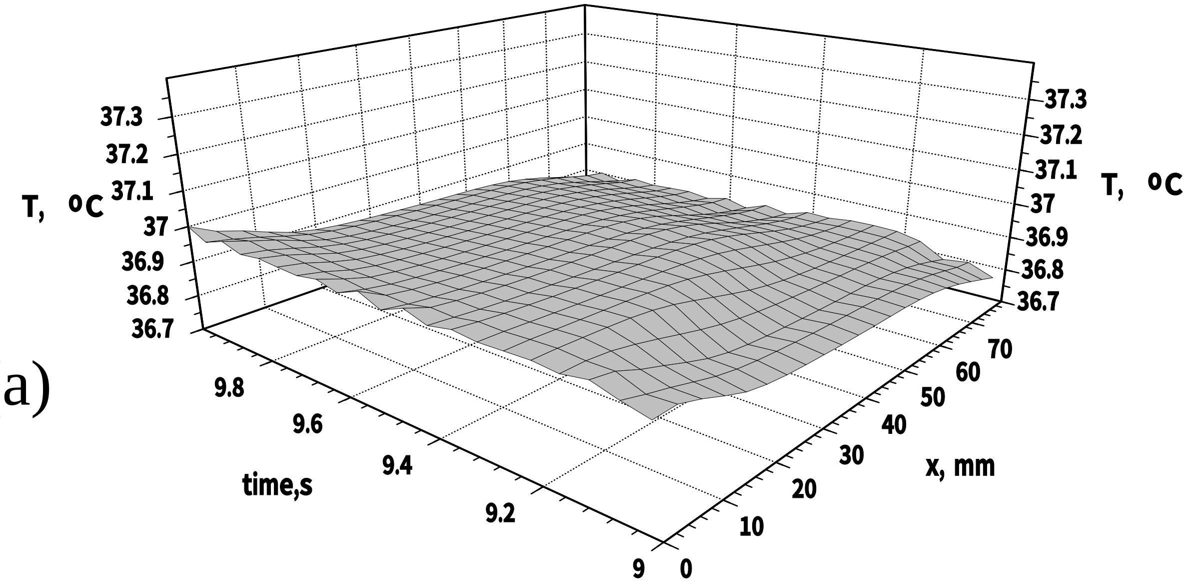
Arteries with a bifurcation point



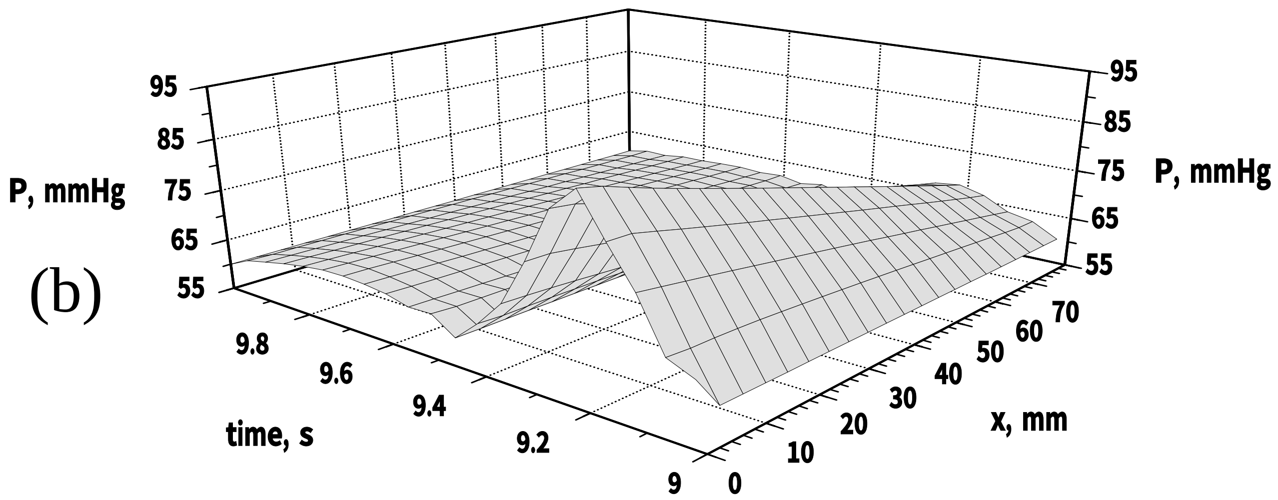


The temporal variations of arterial temperature (a) and flow rate (b) at different inflow conditions

(a)

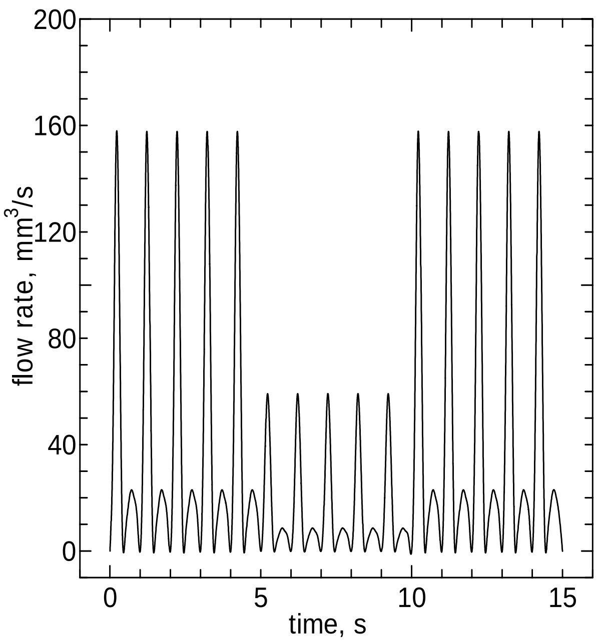


(b)

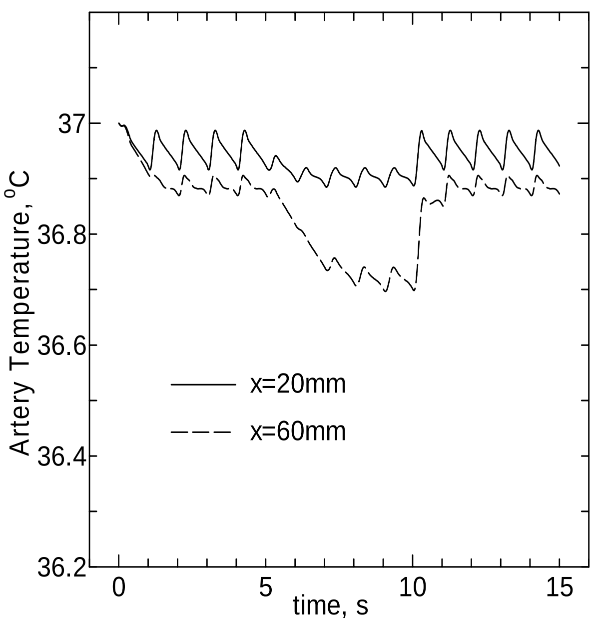


Temperature (a) and pressure (b) of the artery as functions of x and t during one period

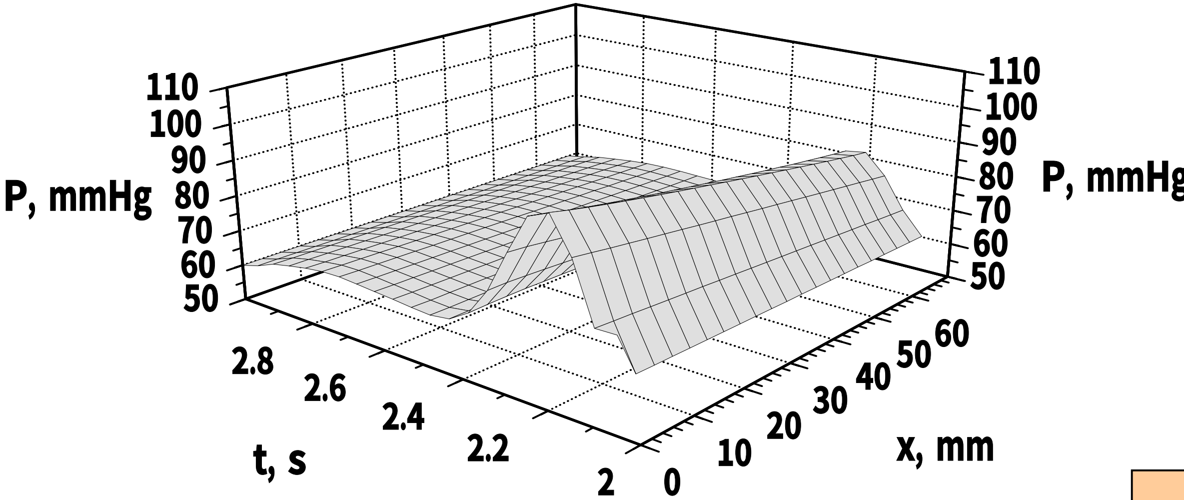
(a)

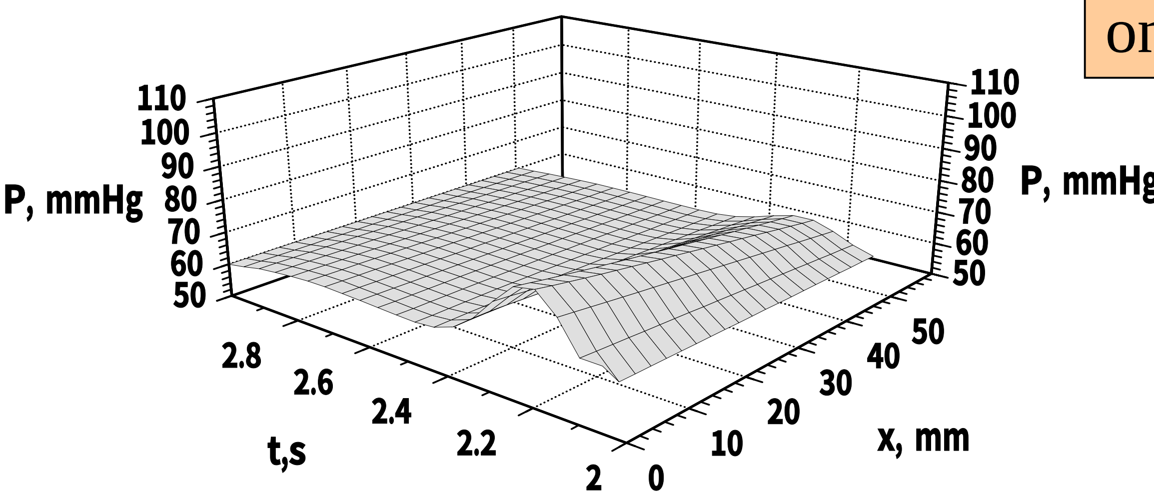


(b)

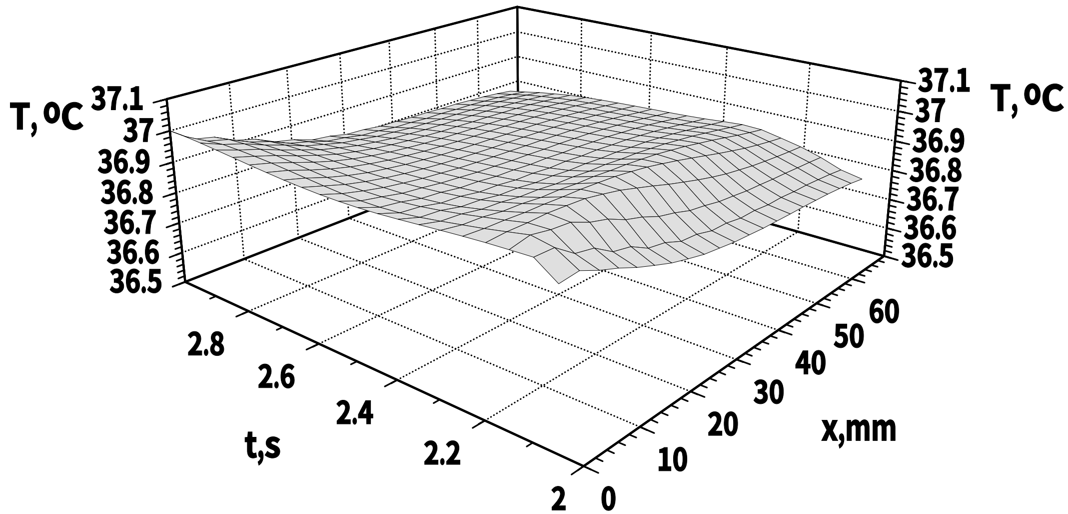


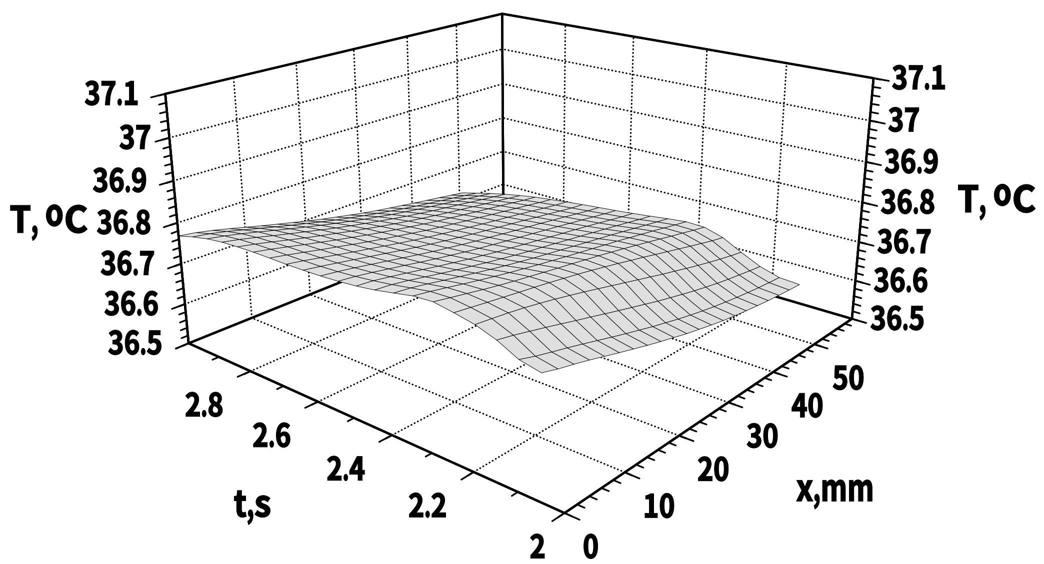
Temporal variations of the flow rate (a) and arterial temperature (b) in the transient state



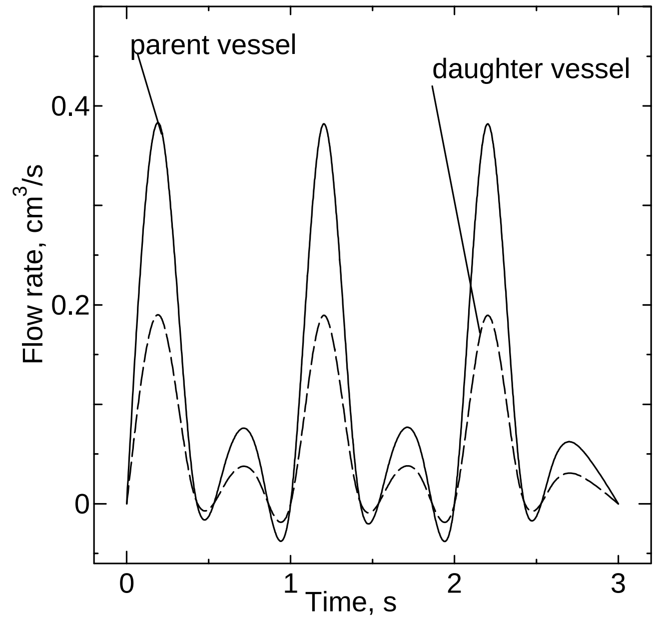


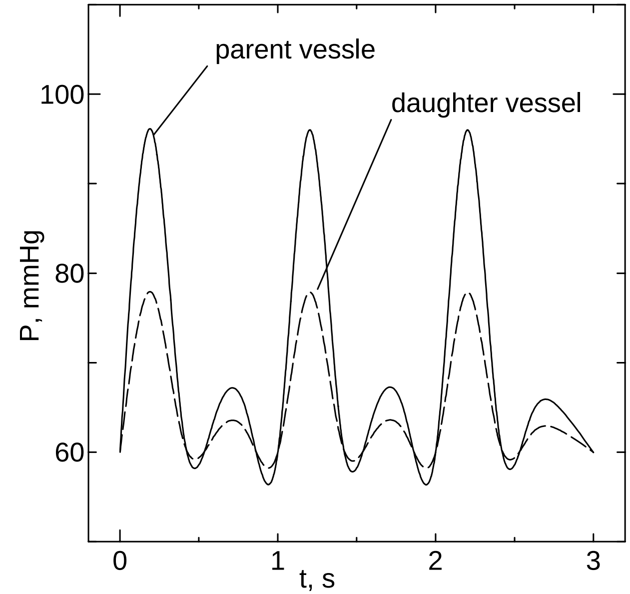
The variation of the pressure in the parent and daughter vessel for one period

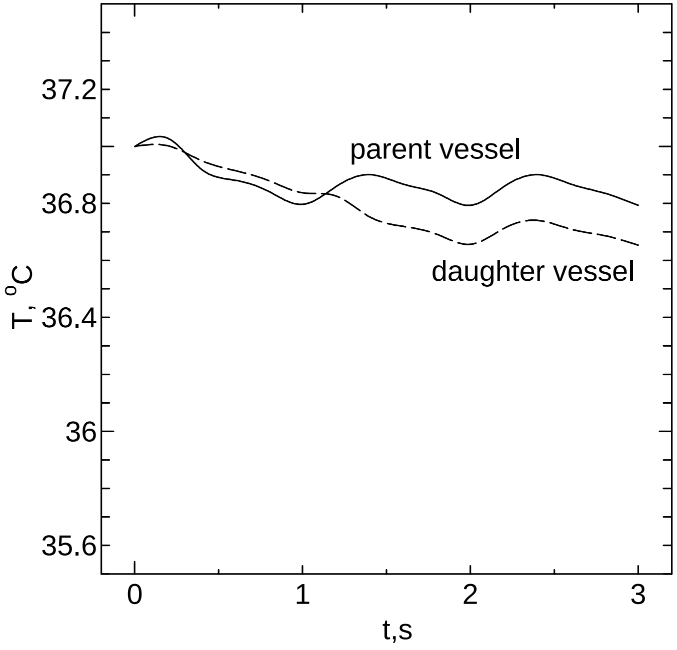




The temperature variation in parent and daughter vessel for one period







The Variations of pressure, flow rate, and temperature with time

Concluding Remarks

- A one-dimensional thermo-fluid model of the blood vessel is presented. The waveform flow rate, transmural pressure, temperature, and elasticity of the vessel are each considered in this model

- The results of the initial study are presented and it is found that the arterial temperature closely followed the change of blood inflow rate

- Further studies are needed to build up a computer model for the human systemic circulation that includes the systemic arterial circulation and the venous return flow circulation.